Journal of Applied Mechanics and Technical Physics, Vol. 47, No. 5, pp. 757–761, 2006

PREDICTION OF FRACTURE IN THE VICINITY OF FRICTION SURFACES IN METAL FORMING PROCESSES

S. E. Alexandrov and E. A. Lyamina

UDC 539.374

It is shown that the use of a fracture criterion containing a characteristic length of the flow region makes it possible to further develop the theory of fracture in the vicinity of the maximum friction surfaces in metal-forming processes, with allowance for an infinite equivalent strain rate arising near such surfaces. A model of perfectly plastic rigid solids is considered in formulating the criterion. It is noted that the approach can be extended to more complicated models of plastic solids.

Key words: *fracture, friction, singularity, plasticity.*

In metal-forming processes, the fracture often occurs in the vicinity of a friction surface. It should be noted that the conditions of metal deformation in a thin layer near the friction surface are substantially different from the deformation conditions in the bulk of the material. This fact is confirmed by formation of a material layer with specific properties in the vicinity of the friction surface [1–3]. Such a distribution of material properties can be theoretically described on the basis of the singular behavior of the velocity field near the maximum friction surface (surface where the specific friction forces are equal to the shear yield point) [4]. It was found [4] that the equivalent strain rate $\xi_{\text{eq}} = \sqrt{2/3}\,(\xi_{ij}\xi_{ij})^{1/2}$ in the model of a perfectly plastic rigid material is inversely proportional to the distance from the friction surface, and a strain-rate intensity factor (coefficient at the main term in the expansion of the equivalent strain rate near the friction surface) was introduced. Based on the strain-rate intensity factor, an approach was proposed in [5] for estimating the thickness of the layer of intense strains near the friction surfaces. The equivalent strain rate is also part of most empirical fracture criteria used for predicting fracture in metal forming [6–8]. The direct application of these local fracture criteria, however, is impossible because they predict fracture for all process conditions with an infinite equivalent strain rate on the friction surface. The use of the concept based on the strain-rate intensity factor (following [5]) for estimating the thickness of the layer of intense strains seems also unreasonable because the fracture process considered is caused by damage accumulation and depends on hydrostatic stress. It seems more reasonable to use nonlocal criteria of fracture containing the characteristic length of the flow region. One such criterion proposed in [9] for the description of fracture near the crack tip generalizes the known local criterion under the assumption that the latter should be satisfied in a certain finite region near the crack tip. A similar procedure can be used to describe fracture in the vicinity of friction surfaces in metal-forming processes under the assumption that a certain local criterion of fracture is satisfied in a certain region near the friction surface. In particular, one of the most frequently used local criteria of fracture is based on the use of the workability diagram and is described by the equations

$$\varepsilon_{\text{eq}}^f = \Phi(\beta_{a\nu}), \qquad \beta_{a\nu} = \frac{1}{\varepsilon_{\text{eq}}} \int_0^t \beta\xi_{\text{eq}}\,dt, \tag{1}$$

where ε_{eq} is the equivalent strain, $d\varepsilon_{\text{eq}}/dt = \xi_{\text{eq}}$, $\varepsilon_{\text{eq}}^f$ is the equivalent strain at fracture, t is the time, and $\beta = 3\sigma/\sigma_{\text{eq}}$, where σ is the mean stress and $\sigma_{\text{eq}} = \sqrt{3/2}(s_{ij}s_{ij})^{1/2}$ is the equivalent stress (s_{ij} are the components of the stress

Institute for Problems in Mechanics, Russian Academy of Sciences, Moscow 119526; lyamina@mail.northnet.ru; sergeyaleksandrov@yahoo.com. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 47, No. 5, pp. 169–174, September–October, 2006. Original article submitted July 20, 2005; revision submitted November 15, 2005.

deviator). The function $\Phi(\beta_{a\nu})$ is determined from the experiment, and integration in (1) is performed for a material point. As $\xi_{\rm eq} \to \infty$ on the friction surface, $\varepsilon_{\rm eq}$ tends to infinity at material points moving along this surface, and the condition $\varepsilon_{\rm eq} > \varepsilon_{\rm eq}^f$ is satisfied for typical functions $\Phi(\beta_{a\nu})$ at the beginning of the deformation process, which means instantaneous fracture independent of other conditions of deformation. It should be noted, however, that sometimes the functions $\beta_{a\nu}$ introduced do not predict fracture for any level of equivalent strain in a certain range of $\beta_{a\nu}$. (This case is not considered in the present paper.)

A nonlocal criterion of fracture can be introduced under the assumption that $\varepsilon_{\rm eq} \geqslant \varepsilon_{\rm eq}^f$ in a certain region of a prescribed size. We assume this inequality to be valid for $s \leqslant s_c$ (s is the distance between the friction surface and the point considered). Then, despite the condition $\varepsilon_{\rm eq} \to \infty$ as $s \to 0$, origination of fracture depends on the process conditions. Currently, there are no experimental data for determining the value of s_c. It is known, nevertheless, that the thickness of the layer of intense strains near the friction surface is approximately equal to 1/10 of the characteristic length of the flow region [3]; in the first approximation, therefore, we can assume that the value of s_c is also equal to 1/10 of the characteristic length of the flow region.

In many cases, an approximate analysis of drawing and extrusion processes is based on solving the problem of a radial flow of a plastic mass in an infinite plane or axisymmetric channel (see, e.g., [10–12]). In particular, solutions for a perfectly plastic rigid material were obtained in [10, 13]. The solution of [10] was used in the present work to determine the conditions of fracture origination during drawing of an axisymmetric bar.

The stresses in a spherical coordinate system $r\theta\varphi$ are determined by the relations

$$\sigma_{\theta\theta} = \sigma_{\varphi\varphi} = \sigma_{rr} - \sqrt{3}\,k\cos\gamma = -kb\ln\frac{r}{r_0} - 3k\int\limits_0^\theta \sin\gamma\,d\theta, \qquad \sigma_{r\theta} = k\sin\gamma. \tag{2}$$

Here k is the shear yield point, b and r_0 are constants found from the boundary conditions, and $\gamma(\theta)$ is a function determined by the equation

$$\cos\gamma\,\frac{d\gamma}{d\theta} + \sin\gamma\cot\theta + 2\sqrt{3}\,\cos\gamma = b. \tag{3}$$

As $\sigma_{r\theta} = 0$ at the axis of symmetry and the maximum friction law $\sigma_{r\theta} = k$ is satisfied on the channel wall where $\theta = \alpha$ (Fig. 1), it follows from Eq. (2) that the solution of Eq. (3) should satisfy the conditions

$$\gamma = 0 \qquad \text{for} \quad \theta = 0; \tag{4}$$

$$\gamma = \pi/2 \qquad \text{for} \quad \theta = \alpha. \tag{5}$$

The solution of Eq. (3) and conditions (4) and (5) determine the function $\gamma(\theta)$ and the dependence of b on α found numerically. Knowing $\gamma(\theta)$ and using Eq. (2), we can determine r_0 from the condition that the horizontal load equals zero for $r = R$ (Fig. 1). This condition corresponds to the drawing process without reverse tension and has the form

$$\int\limits_0^\alpha (\sigma_{rr}\cos\theta - \sigma_{r\theta}\sin\theta)\sin\theta\,d\theta = 0. \tag{6}$$

(The value of r_0 for given parameters of the process was found numerically.) As we have $\sigma_{\rm eq} = \sigma_Y = \sqrt{3}\,k$ for a perfectly plastic material that obeys the Mises yield criterion, the parameter β is found from Eq. (2) in the form

$$\beta = \frac{\cos\gamma}{3} - \frac{b}{\sqrt{3}}\ln\frac{r}{r_0} - \sqrt{3}\int\limits_0^\theta \sin\gamma\,d\theta. \tag{7}$$

Thus, with allowance for the solutions of Eqs. (3) and (6), the parameter β is a known function of the variables θ and r for given geometric parameters of the process.

The radial velocity is determined by the expression [10]

$$u = -\frac{B}{r^2}\exp\Big(-2\sqrt{3}\int\limits_0^\theta \tan\gamma\,d\theta\Big), \tag{8}$$

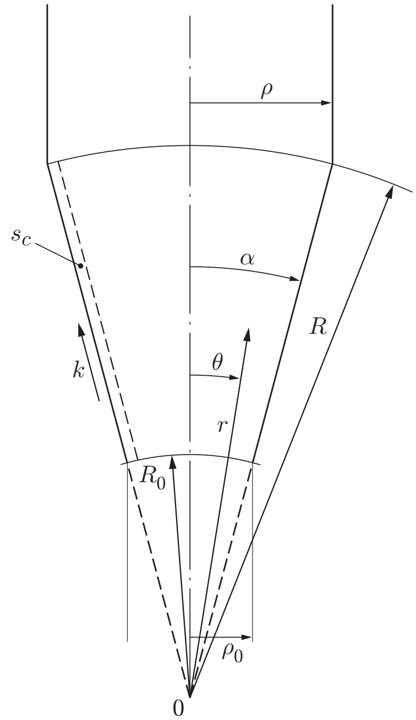

Fig. 1. Geometry of the drawing process.

where B is a constant, which can be determined for a given velocity of the rigid area above the surface $r = R$ (Fig. 1). The value of B, however, does not affect the fracture process under consideration. As the flow is steady and the component u is the only velocity component that differs from zero, we have

$$\varepsilon_{\rm eq} = -\int_r^R \xi_{\rm eq} \frac{dr}{u}. \tag{9}$$

Using Eq. (8), we obtain the expression $\xi_{\rm eq}/u$ in the form

$$\frac{\xi_{\rm eq}}{u} = -\frac{2}{r\cos\gamma}. \tag{10}$$

Substituting Eq. (10) into Eq. (9), we find

$$\varepsilon_{\rm eq} = \frac{2}{\cos\gamma} \ln\frac{R}{r}. \tag{11}$$

It follows from relation (11) and condition (5) that $\varepsilon_{\rm eq} \to \infty$ in approaching the friction surface. Substituting Eqs. (7), (10), and (11) into Eq. (1) and integrating the latter, we can find the distribution of $\beta_{a\nu}$. According to the experimental data of [3], the value of s_c for an aluminum alloy is approximately 1/10 of the characteristic length of the flow region; therefore, we assume in the case considered that $s_c = r\alpha/10$. Then, using the formulated criterion of fracture for the value of s_c mentioned above and the prescribed function $\Phi(\beta_{a\nu})$ and assuming that

$$\varepsilon_{\rm eq} = \Phi(\beta_{a\nu}) \tag{12}$$

for $\theta = 9\alpha/10$ and $r = R_0$, we can determine the process parameters at which fracture begins.

For steel (with the chemical composition containing 0.32–0.39% C, 0.15–0.35% Si, and 0.5–0.8% Mn), the function $\Phi(\beta_{a\nu})$ has the form [14]

$$\Phi(\beta_{a\nu}) = 0.639 - 0.568\beta_{a\nu} + 0.138\beta_{a\nu}^2. \tag{13}$$

With the use of the numerical solution obtained and relations (12) and (13), we determined the critical value of $q = \rho/\rho_0 = R/R_0$ at which fracture occurs as a function of the angle α, which is plotted in Fig. 2 [ρ and ρ_0 are

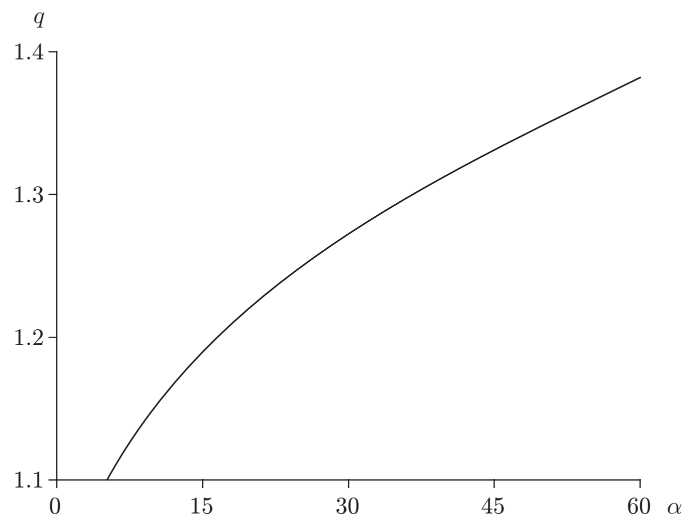

Fig. 2. Ratio of the radii of the input and output sections of the bar at which fracture occurs versus the die angle α.

the bar radii at the input and output of the die, respectively (see Fig. 1)]. The values of the radii ρ and ρ_0 do not affect the fracture process for a chosen solution and fracture criteria. This is a consequence of the fact that the value of s_c depends on the characteristic length of the flow region.

The subject of the study in the present paper is a perfectly plastic rigid material. The proposed approach (or another nonlocal criterion of fracture), however, may also have to be used for more complicated models of materials. The theory of the work-hardening plastic solids widely employs the dependences of equivalent stress on equivalent strain, which contain the saturation stress (see, e.g., [15]). As it follows from solving particular problems [16], if such dependences are used, the equivalent strain rate can tend to infinity on the maximum friction surface. Singular velocity fields arise near the maximum friction surfaces if certain models of a plastic anisotropic material are used (see, e.g., [17]). Models of viscoplasticity and superplasticity containing the saturation stress are also available [18, 19]. In these cases, probably, the equivalent strain rate can tend to infinity on the maximum friction surface.

This work was supported by the Russian Foundation for Basic Research (Grant No. 05-01-00153), by INTAS Foundation (Grant No. 04-83-2723), and by the Foundation "Leading Scientific Schools in Russia" (Grant No. NSh 1849.2003.1).

REFERENCES

1. S. I. Gubkin, *Plastic Deformation of Metals* [in Russian], Vol. 3, Metallurgizdat, Moscow (1961).
2. Y.-T. Kim and K. Ikeda, "Flow behavior of the billet surface layer in porthole die extrusion of aluminium," *Metallurg. Mater. Trans.*, **31A**, No. 6, 1635–1643 (2000).
3. T. Aukrust and S. LaZghab, "Thin shear boundary layers in flow of hot aluminium," *Int. J. Plast.*, **16**, No. 1, 59–71 (2000).
4. S. Alexandrov and O. Richmond, "Singular plastic flow fields near surfaces of maximum friction stress," *Int. J. Non-Linear Mech.*, **36**, No. 1, 1–11 (2001).
5. S. E. Alexandrov, R. V. Goldshtein, and E. A. Lyamina, "Developing the Concept of the Strain Rate Intensity Factor in Plasticity Theory," *Dokl. Ross. Akad. Nauk*, **389**, No. 2, 180–183 (2003).
6. V. L. Kolmogorov, *Plasticity and Fracture* [in Russian], Metallurgiya, Moscow (1977).
7. A. G. Atkins, "Fracture in forming," *J. Mater. Process. Technol.*, **56**, 609–618 (1996).
8. M. A. Shabara, A. A. El-Domiaty, and M. A. Kandil, "Validity assessment of ductile fracture criteria in cold forming," *J. Mater. Eng. Perform.*, **5**, 478–488 (1996).
9. D. M. Norris, J. E. Reaugh, B. Moran, and D. F. Quinones, "A plastic-strain, mean-stress criterion for ductile fracture," *Trans. ASME, J. Eng. Mater. Technol.*, **100**, No. 3, 279–286 (1978).

10. R. T. Shield, "Plastic flow in a converging conical channel," *J. Phys. Mech. Solids*, **3**, No. 4, 246–258 (1955).
11. N. Cristescu, "Plastic flow through conical converging dies, using a viscoplastic constitutive equation," *Int. J. Mech. Sci.*, **17**, 425–433 (1975).
12. D. Durban and M. E. Mear, "Asymptotic solution for extrusion of sintered powder metals," *Trans. ASME, J. Appl. Mech.*, **58**, No. 2, 582–584 (1991).
13. V. V. Sokolovskii, "Plane and axisymmetric equilibrium of a plastic mass between rigid walls," *Prikl. Mat. Mekh.*, **14**, 75–92 (1950).
14. S. E. Alexandrov, D. Vilotich, R. V. Gold'stein, and N. N. Chikanova, "On determining the workability diagram," *Izv. Ross. Akad. Nauk, Mekh. Terdv. Tela*, No. 4, 141–149 (1999).
15. J. G. Sevillano, P. van Houtte, and E. Aernoudt, "Large strain work hardening and textures," *Prog. Mater. Sci.*, **25**, No. 1, 69–112 (1981).
16. S. Alexandrov and O. Richmond, "Frictional effects in the modified Couette flow of solids," in: *Proc. of the 8th Int. Conf. on Metal Forming* (Cracow, Poland, September 3–7, 2000), Balkema, Rotterdam (2000), pp. 723–728.
17. I. F. Collins and S. A. Meguid, "On the influence of hardening and anisotropy on the plane-strain compression of thin metal strip," *Trans. ASME, J. Appl. Mech.*, **44**, No. 2, 271–278 (1977).
18. L. Tashman, E. Masad, D. Little, and H. Zbib, "A microstructure-based viscoplastic model for asphalt concrete," *Int. J. Plasticity*, **21**, No. 9, 1659–1685 (2005).
19. O. M. Smirnov, *Metal Working in a Superplasticity State* [in Russian], Mashinostroenie, Moscow (1979).